EUROPEAN ELECTRON INDUCTION ACCELERATORS

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SUMMARY

Betatrons, induction electron accelerators for the production of high energy x-radiation, were under development in Germany during the war and are also to some extent at present. This work is reviewed for the most part and details are given on the constructional features of 6 and 15 million volt betatrons and on the theory and design of 15 and 200 million volt betatrons. The smaller units, especially the Siemens 6 Mv are quite successful and more advanced than comparable American units. While no large machines were actually built the projected 200 Mv Wideroe design introduces new features of value. Betatron patent ideas are reviewed.

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U.S. NAVAL TECHNICAL MISSION IN EUROPE

TABLE OF CONTENTS

List of Illustrations

Fig. No.

EUROPEAN ELECTRON INDUCTION ACCELERATORS

I. Introduction.

A. History of Betatron Developments on Continent.

The first work on the subject of electron induction acceleration which appeared in Europe was the doctoral dissertation(1) of Rolf Wideroe of Bergen, Norway, which appeared in 1927 and which was published in print the following year(2). This was coincident with the American Slepian patent (U.S. Pat. 1,645,304 Oct. 11, 1927), and some experiments tried at the Carnegie Institute in Washington. These works lacked requisite details on principles for the successful production of high speed electrons. Another work appeared in England in 1929 by E.T.S. Walton whose work was instignated by a suggestion of Lord Rutherford. This also was incomplete.

Developments on the continent have been based on Wideroe's work. The basic ideas were by Steenbeck in his patents of 1937 (DRP 656,378 and 698,867)(U.S. Patent 2,103,303 Dec. 28, 1937) in which the necessary properties of the magnetic accelerating and guiding fields are folmulated.

An induction electron accelerator was built on these principles and tried with success, it is claimed, although the intensities of radiation produced were only of cosmic ray intensity order. Following Kerst's successful production of 2.3 Mv gamma radiation(3) in intensities equivalent to that of milligrams of radium in 1941, Steenbeck(4) published a belated note on his work in 1943.

This work of Kerst's was a great incentive to continental workers interested in this field, principally Steenback and Wideroe.

It has been learned that only the latter has been very active however, Steenbeck having been engaged in other matters during the course of the war.

(1) Dissertation Tech. Hochshule Aachen, 29 October 1927
(2) Arch. Electrotechnik 21 (1928) pp 387.
(3) Phys. Rev. 60 (1941) P. 47.
(4) Naturwiss. 19/20 (1943) p. 234.

Wideroe and the group that came to be associated with him in the war-time German betatron work were not in sympathy with the Nazi cause, and were pursuaded to continue their work for purely scientific considerations.

The betatron development was carried out by three separate groups. One was the Megavolt Versuchsanstalt (MVA) or Megavolt Research association which worked under the guidance of Dr. Wideroe. Siemens and AEF also had projects. MVA laid plans for a 15 Mv and a 200 Mv accelerator. The former was begun early in 1944 and brought to successful completion in the fall of the same year. The design of the 200 Mv accelerator was completed about this time and the construction work turned over to the Brown Boveri firm in Heidelberg. Recent investigation showed that MVA had no further connection with this project.

The basic ideas underlying the design of these two induction accelerators were published by Wideroe in two papers(1). These discuss the special problems involved such as axial and radial stabilization by properly designed guiding fields, injection and extraction of electrons, the useful load of the apparatus, the maximum electron filling of the acceleration chamber, dependence on injection voltage (using auxiliary lens system), the stabilization of electronic orbits with electrical lenses, especially for high radiation energies, the effects of gas molecules, technical transformer construction details for high energy production, and finally the fields of application of the betatron: cancer treatement, radiography, and nuclear physics.

In collaboration with the design work of Wideroe, a considerable amount of theoretical work has been carried out by Touschek. This is known to have been of invaluable aid in the development of the 15 Mv accelerator. Further theoretical work has also been done by Touschek in the starting of electrons in the accelerator. Some of the work is along the lines initiated by Kerst and Serber which were known to Touschek. Concerning this latter work, it may be added that the discrepancies between the Kerst-Serber theories and experimental results can now be explained.

(1) Wideroe, Part 1, Arch F. Elektrotechnik 37 (1943) 391-408 Part II remains in proof form.

These may be of use in further electron injection studies.

B. Note on Basic Patents in Betatron Field.

The earliest existent patent is that of Slepian, U.S. No. 1, 645, 304, which was filed April 1, 1922, Ser. No. 548, 636 and granted Oct. 11, 1927. An application was filed in Germany Feb. 16, 1923 and granted May 23, 1929. (Published June 8) DRP No. 477, 498. Considerable difficulty was had in obtaining the German patent because Dr. Schmid of the German Patent Office contended that the application was based on an impossibility. In support of this Prof. Abraham's texts were cited to contend that a magnetic field could only guide an electron but not accelerate it. Reply was made by the Westinghouse firm that the idea was misunderstood and that no such claim as stated by Dr. Schmid was implied. After more correspondence it was reluctantly admitted that there was a possibility of electron acceleration and a patent was finally granted. Dr. Hoffman, Chief of Patent Dept., Siemens-Reiniger, Erlangen, gave supporting comment in review of the patent in 1927, designating it as not involving infringement of any Siemens claims. In brief the patent involves the use of tangential electric fields resulting from the change of a magnetic flux to accelerate the electrons in a circular orbit. A yoke similar to that employed at present was proposed to provide the change in field. The patent does not give the conditions for the stabilization of electron orbits.

The second basic patent is that of Steenbeck, DRP 656,378 of April 1, 1933, followed by DRP 698,867 of March 7, 1935. Steenbeck was acquainted with the work of Wideroe whose thesis appeared in 1927 and wrote into the patent the necessary conditions for the production of a stable (radially) orbit in the magnetic acceleration process which was known to Wideroe. Steenbeck also supplied the condition in which electrons could be magnetically focused to stabilize them in the axial direction. This patent was published Dec. 6, 1940. An American license for its use by the American General Electric Co. was requested from Siemens shortly before the outbreak of war in 1941. The Steenbeck patent was not sufficient to construct a successful induction accelerator. The necessary condition for the introduction of electrons was first stated in the patent of Kerst, an early application of which was made and granted in France, French Patent No. 878, 769, Application Jan. 14, 1942, Granted Oct. 26, 1942, and published Jan. 29, 1943 by the Cie Francaise Pour L'Exploitation des Procedes Thompson Houston which is the French licensee of the American General Electric Co.

The announcement of successful production of accelerated electrons and X-rays by Kerst[1] and subsequent publications[2] stimulated an interest in the induction method of electron acceleration on the continent and late in Dec. 1941, designs were under way at Siemens-Reiniger for construction of an electron induction accelerator. Dr. Steenbeck gave the basic specifications which were incorporated into a design at Siemens and discussed in conference on 15 December 1942 by him with Dr. Ganswindt, Mr. Kurt Bischoff, Dr. J. Patzold and Dr. K. Gund of Siemens. At this time appeared also a design of Dr. Gund proposing an induction accelerator operated with a fixed magnetic guide field, surrounding an alternating magnetic field passing through a central core, a scheme originating from a paper by W.W. Jasinsky[3] in which the conditions of the field are however erroneously set forth. The design of Gund corrects this error and introduces a scheme for injecting electrons from a ring cathode form, flanked by electrostatic guide fields, so that it is possible to accelerate the electron in spiral shaped courses rather than in a fixed equilibrium orbit. The advantages and the basic facts which require consideration in further patent innovations may be summarized: The accelerating magnetic field is contained in an unbroken iron circuit. A complete half-cycle of operation is possible. This appears in other designs too, as pre-magnetization schemes. The electrons may be introduced into the accelerator for a large part of the period. This obviates in part the difficulties of injecting electrons in the short time available while the acceleration magnetic field is passing through its zero point and also the difficulty of increasing the electron orbit acceleration in the initial stages due to space charge effects. A patent application was entered by Siemens-Reiniger covering these features July 28, 1942 (Aktenzeichen S 151,465 VIII c/21g).

(1) Kerst D. Phys. Rev. 58 (1940) p. 841.
(2) Kerst D. Phys. Rev. 59 (1941) p. 110 and 60 (1941) pp. 47-53.
(3) W.W. Jasinsky. Arch. F. Elektrotch. 30 (1930) pp. 590-603.

This was rewritten to include a number of other features about which more will be said in a later section of this report. The revision appears in a second patent application dated Aug. 8, 1942 (Aktenzeichen s. 151, 603 VIII c/21g). Suffice it to say at this point that the applications include the use of both time-variant and time-invariant guide fields, an electron acceleration space divided into two portions, (an inner stabilizing region and an outer region in which a stable orbit is produced) means to decrease or increase the guide field strength for electron orbit contraction and expansion, and schemes to release electrons from the accelerator in a beam by means of deflection plates.

The construction of the pole pieces for induction acceleration is touched upon to some extent in the patent of Steenbeck mentioned above. Yoke construction involves few novelties and follows for the most part transformer construction principles. Its use is probably covered in the Slepian patent. The application of radial construction in the laminated pole pieces is likewise older than any betatron patents. This feature appears in the Berry transformer of 1906 and has been further developed for transformer construction by the Brown, Boveri who furthermore have incorporated welding as a means of reinforcement.

This type of construction is suitable for the construction of large induction accelerators and brings with it the problem of cooling large radially laminated cores. A recent patent application in this direction is that of the Brown Boveri Co. assignor Dr. Helmut Bocker in which a radially laminated core is especially made with helical ducts for cooling a radial core with air without introducing dissymmetries which would be disturbing in the field distribution at the cylinder surface of the core (Cf. BBC Aktennotiz and Sketch, K 363, 607, Aug. 2, 1945).

II. Activities of the Megavolt Versuchsanstalt (Mégavolt research Establishment) of Wrist near Erlangen, Germany.

A. Details of the 15 Mv Induction Electron Accelerator.

1. Description.

The choice of the electronic voltage to be generated was dictated by the following considerations: Theoretical curves in which the absorption coefficient of the x-radiation is plotted against the electronic energy show a minimum as a result of the decreasing contribution of the Compton scattering and the increasing contribution of the absorption due to the formation of electron pairs. These minima lie in the region of 3-10 Mv and it is therefore of importance to choose a peak value of 15 Mv so that the effective energy will be about 7-8 Mv in the desired range for radiographic purposes. In copper or steel for instance the absorption coefficient for 15 Mv radiation is about 1/6 of that for 150 KV X-rays. This provides a theoretical six-fold increase in radiographic range. The medical side of the requirements are a little different. The absorption curves of water and light elements have their minima at fairly high energies, say 20-50 Mv. For medical purpose, it is necessary, in order to treat deep-lying tissues, to insure the penetration of the radiation so that the deep-lying tissue is reached without excessive absorption in the intervening layers. At 15 Mv, the absorption in water is about 1/3 of the absorption of 1 Mv radiation. Even with this advantage the betatron radiation is not altogether satisfactory from the standpoint of the depth dose administered to points beyond the locality to be treated. For this purpose, a problem which is still under way is the utilization of the betatron for producing high energy electron beams in which the dissipation of energy by absorption in human tissue can be localized much more effectively.

(a) Construction Details, Core.

The radius of the electron equilibrium orbit was chosen as r = 14 cm. The diameter of the induction pole was chosen as 220 mm. (radius, 11 cm) and the air gap in the poles was made 2 cm. The average separation of the guide poles was chosen as 6.0 cm. and the contour of the guide pole faces was determined from magnetic measurement and also from measurements by means of the electrolytic trough. This process was continued until the guide fields had just the properties required. The guide poles were so arranged with respect to the remainder of the apparatus that as a result of the saturation of the guide field the guide field does not increase in proportion to the accelerating field from a certain point in the cycle onwards. The approximate data for the main components (which are slightly above the initial estimates of Dr. Wideroe) are:

Iron Weight	1000 Kg	(2200 lb.)
Copper weight	200 Kg	(440 lb.)
Total losses	2.5. KW at	50 cycles/sec
Dead load	250 KVA	
Driving voltage	6 KV	

The overall dimensions and details of core and pole assembly are shown in figure 1. The yoke is constructed from laminated iron of thickness equivalent to 29 gauge, .014 inches stock of approximately 1/2 watt per lb. core loss, (10,000 lines, 60 cycles). The lamination stack is not divided, i.e. no provision is made for cooling the core by air ducts. It was found that heating was not excessive. The vertical members of the yoke, in contrast to American design, extend the full height of the apparatus, and the cross or horizontal members are held between these by a special arrangement of clamping frames. This produces four vertical "shim" gaps. On the upper side these have means for adjusting the height and level of the upper transverse member (yoke). This carries the accelerating pole by means of six bolts circularly arranged. By means of a supporting ring, it also carries the guide pole and coil boxes. The yoke laminations are clamped by transverse bolts through insulating tubes.

(b) Acceleration chamber.

The acceleration chambers used were of glass made by the glass blowing laboratory at the firm of C.H.F. Mueller. These have a somewhat elliptical cross section. Graphite and silver coatings were produced in the inside and the tubes have either two or three radial openings with stems on the periphery for evacuation on mounting of the electron injection assembly. Vacuum was produced by an oil diffusion pump backed by a three stage mercury diffusion pump. A liquid air trap separates the two. The mercury diffusion pump is backed in turn by a rotary oil pump. This unnecessary complication of pumps was due to the scarcity of available and usable equipment.

The vacuum was guaged by means of a Phillips vacuumeter between the two diffusion pumps and by means of an ionization guago sealed to the acceleration chamber at the injection stem.

It was noted that operation was not possible at pressure above 10^{-5} mm Hg and steady yields were obtained only at 10^{-6} mm.

The electron gun or injector is shown in figure 2 and follows in principle the design introduced by Kerst. It consists of a tungsten filament (oxide coated electron sources were also used) surrounded by a semi-cylindrical

Wehnelt cylinder (Faraday cage) and of two anode plates insulated from each other. The cage serves, on application of a potential between itself and the filament, to concentrate the electron stream. The split anode, on the two halves of which different potentials could be applied has, ignoring for a moment the necessary acceleration of electrons, a purpose similar to the deflection plate of a cathode ray tube. It is thus possible by applying various potentials to these plates to give the electrons various initial projection angles in the plane of the orbit. The necessary correction of the electron beam in the vertical direction is obtained by a small rotation of the whole injector system or stem on its ground glass joint. A small bar of tungsten is fastened to the innermost anode plate with its long axis vertical to serve as the target for x-ray production.

The filament is heated continuously. The injection voltage impulse is applied only for a short time at the beginning of the acceleration period by means of a special arrangement to be described below, partly to charge the entire injector structure to as high a voltage as possible and partly to avoid filling the electron accelerator chamber with stray electrons.

(c) Connections.

Figure 3 shows the entire diagram of connections. Alternating current of 6 K.V. is generated by a regulating transformer and supplied to the driving circuit in which the dead load of the circuit is compensated by a capacitor bank, the capacity of which may be varied between limits. The injection voltage is produced by H.V. equipment from which the potential may be applied at the proper instant near the beginning of the acceleration period by means of a special grid-controlled high voltage rectifier tube. The driving circuit of the betatron passes through a small utility transformer which in turn operates a peaking transformer for the production of a sharp positive pulse to trip or remove momentarily the large negative potential on the grid of the control valve.

In earlier experiments, a mechanical contactor driven by a synchronous motor was used to apply the tripping pulse but this gave much trouble due to fouling of contacts. The circuit diagram also shows a 5 KV constant potential circuit used for applying the deflection voltages to the split anode, the innermost plate of which is connected together with the acceleration chamber coating to ground.

2. Operation.

The detection of radiation at the outset of operations is carried out by means of a counter tube. This however soon becomes useless due to jamming at higher intensities because of the combined effects of emmission of radiation in small pulses and lack of resolving power of the counter tube and its circuit. Ionization chamber measurements are then resorted to for almost all radiation measurements. The counter tube is retained in use for indicating the point in the cycle where radiation is emitted by passing the output of the counter tube circuit into a cathode ray oscillograph tube in which the horizontal or time base is driven by the A.C. driving voltage.

In operation the vacuum is checked, the driving circuit is energized and the phase of the injection pulse is determined and corrected by means of the phase shifting device. On starting the counter plus cathode ray apparatus, a radiation output is usually observed. By rotation of the ground joint of the injector, by application of a suitable deflector voltage, by changing the position of the acceleration chamber in the machine and also by adjustment of the air gap in the guide field, it is possible gradually to bring about an optimum yield. Then by keeping these variables fixed quite reproducible results could be depended on, which were then dependent only on the vacuum conditions. Measurements with the ionization chamber could then be carried out.

To check the quality of radiation produced by the machine various methods were resorted to: (a) Observation of the counter pulse in the oscillograph allowed the time of acceleration of the electrons to be estimated and deductions of their maximum energy to be made. This in general was around 13-15 Mv. (b) by absorption measurements on the radiation, which however are somewhat uncertain because of the shallow minimum in the absorption versus electron energy curve, (c) rapid dying-out of radiation intensity on reducing the amplitude of excitation current and (d) production of artificial radioactivity by irradiation of various materials.

This phenomenon is especially pronounced in the case of silver and copper.

After the expected quality of radiation had been established attention was directed to improving the intensity of the radiation. The goal here was to attain a maximum electron filling of the acceleration chamber such as might be expected from the formula deduced by Wideroe for the effect of space charge.

$$i = \frac{4 \pi^3 \Delta r^3 (1 - n) f^2 r B_{max}}{c^2 (- \partial r)}$$

This formula usually gives a value about ten times larger than what may be obtained experimentally. The experimental result was a radiation current equivalent by radiation to about 1 kg. of radium. It is believed that this discrepancy is due to release of gases by bombardment of the walls of the acceleration chamber and that better results could be obtained only by the use of baked-out and sealed-off acceleration chambers.

The scattered radiation in the vicinity of the machine was noted to be quite high. Even behind the thick legs of the yoke the normal tolerance dose is exceeded in 5 – 10 min.

3. Program for Further Development.

The program of the Megavolt Versuchsanstalt as described in June 1945 was: (a) further work to increase the radiation yield of the above described machine and (b) to build as soon as possible a duplicate of this equipment for use in medical research so as to allow the first unit to be used solely for technical development.

Under (a) it was planned to make further experiments with acceleration chambers fitted with electron lens systems which might, in view of the greater stabilization forces available with their use, bring about an increase in radiation output intensity. It was also planned to study the tangential electron injection techniquie of Wideroe once more because of the possible advantage accruing from the absence of an injector structure in the acceleration chamber; also to study the problem of releasing a beam of electrons from the apparatus which would be very important in any event.

Another intention was the design and construction of a 30 Mv unit for use in nuclear research. This project was to have been undertaken at an early date. Design calculations have been made along with those for the 15 Mv unit and could be used for construction specifications once special electrolytic-trough or magnetic measurements had determined the proper shaping of the guide poles.

It was felt that an apparatus of equal size but operating on the premagnetization scheme of Wideroe would be useful in testing such questions as to whether or not the final energy of the accelerator would be affected by radiation damping as predicted by Iwanenko and Pomerantschuk, which work has been studied further by Touschek. The results obtained with the accelerator should have a decided bearing on the validity of classical radiation theory.

A final project was the construction of a 200 Mv. accelerator using the premagnetization scheme. This was considered of special importance to nuclear physics since new phenomena not observed before could be expected at energies of the order of 150 Mv and higher. The construction of this unit was undertaken by the firm of Brown Boveri and Co. in the fall of 1944. Consultation between M.V.A. and the B.B.Co. produced a number of designs from which a most practical one was selected. This was carried to enough detail so that most of purely transformer construction problems could be regarded as solved. A reconsideration of the entire design in view of the possible effects of radiation damping was under consideration. Since the occupation the work on this project has been dropped and the M.V.A. group has had no further contact with B.B.Co. Here the work had progressed to the point that construction of a full scale sector model of the large machine was to begin in order to make operational tests to study the field distribution in the accelerator and also to study the necessary electrical problems involved in the use of premagnetization.

B. Design Calculations for 15 Mv. Betatron.

In the following the design calculations made by Dr. Wideroe for both the 15 and 200 Mv betatrons previously mentioned will be given in view of their possible value for design and construction of similar equipment.

1. General Formulation of Theory.

To introduce this material the basic theoretical formulas relating the electron energy to the size and the magnetic field strength of the induction accelerator will be reviewed. The basic formula for energy of an electron (relativistic) is

$$W = e.U = m_o c^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \qquad \text{(ergs)}$$

where $\beta = v/c$ and U is electron voltage, e and m the charge and mass of the electron, c the velocity of light and v the velocity of electron. Since only U is of direct interest the energy equation may be simplified by letting $m_o c^2/e = \varepsilon$ giving

$$U = \varepsilon \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) \qquad \text{(A)}$$

Rearranging $\quad (U + \varepsilon)^2 = \dfrac{\varepsilon^2}{1 - \beta^2} = U^2 + 2\varepsilon U + \varepsilon^2$

or $\quad 1 - \dfrac{v^2}{c^2} = \dfrac{\varepsilon^2}{(U + \varepsilon)^2}$

$$v = c\sqrt{1 - \frac{\varepsilon^2}{(U + \varepsilon)^2}} = C - \sqrt{\frac{U^2 + 2\varepsilon U + \varepsilon^2 - \varepsilon^2}{(U + \varepsilon)^2}}$$

$$= C\,\frac{\sqrt{U^2 + 2\varepsilon U}}{(U + \varepsilon)} \qquad \text{(B)}$$

Likewise

$$m = \frac{m_o}{\sqrt{1 - \beta^2}} = \frac{m_o}{\sqrt{1 - \left(1 - \frac{\varepsilon^2}{(U + \varepsilon)^2}\right)}} = \frac{m_o}{\frac{\varepsilon}{(U + \varepsilon)}}$$

(C)

$$m = m_o\frac{(U + \varepsilon)}{\varepsilon} = m_o\,(1 + U/\varepsilon)$$

The energy gained by an electron moving in an electric field

$$dU = E\ ds = E(t)\ v\ dt \quad E\text{: field volts/cm} \tag{D}$$

$$= E(t)\ c\ \frac{\sqrt{U^2 + 2U\varepsilon}}{(U + \varepsilon)}\ dt$$

To solve rewrite $\dfrac{dU \cdot (U + \varepsilon)}{c\sqrt{U^2 + 2\varepsilon U}} = E(t)\ dt$

Note that by letting $(U^2 + 2\varepsilon U) = w$

then $dw = 2(U + \varepsilon)\ dU$

$$\text{Then} \int E(t)\ dt = \frac{1}{2c}\int \frac{dw}{w^{\frac{1}{2}}} = \frac{1}{2c}\ \frac{w^{\frac{1}{2}}}{\frac{1}{2}} = \frac{\sqrt{w}}{c}$$

$$= \frac{1}{c}\sqrt{U^2 + 2\varepsilon U + c'} \tag{E}$$

The e.m.f. gained per revolution is

$$\frac{d\phi}{dt} = 2\pi r\ E(t)$$

$$\int \frac{\Phi}{2\pi r}\ dt = \frac{\Phi}{2\pi r} = \frac{1}{c}\sqrt{U^2 + 2\varepsilon U + C'}$$

From proportionality $C' = 0$ and

$$\Phi = \frac{2\pi r}{c}\sqrt{U^2 + 2\varepsilon U} \text{ or } U^2 + 2\varepsilon U = \left(\frac{c\Phi}{2\pi r}\right)^2$$

Completing the square

$$U^2 + 2\varepsilon U + \varepsilon^2 = \left(\frac{c\Phi}{2\pi r}\right)^2 + \varepsilon^2$$

$$(U + \varepsilon) = \sqrt{\left(\frac{c\Phi}{2\pi r}\right)^2 + \varepsilon^2}$$

$$U = \sqrt{\left(\frac{c\Phi}{2\pi r}\right)^2 + \varepsilon^2} - \varepsilon \tag{F}$$

Letting B_i represent the average induction within the orbit

$$\Phi = \pi r^2 B_i \quad \text{and} \quad U = \sqrt{\left(\frac{c\, r\, B_i}{2}\right)^2 + \varepsilon^2} - \varepsilon \qquad (G)$$

Expansion of first term on right side by the binomial theorem shows that only the first term in the expansion need be retained for good approximation. Then

$$U \cong \left(\frac{c}{2} B_i\, r - \varepsilon\right) = \left[1.5\text{x}10^{-4}(B_i r) - 0.509\right] \text{Mv} \qquad (H)$$

Since U in Mv is $\frac{d\phi}{dt} \cdot 10^{-14}$

$$\text{and } B_i = \frac{2}{c\, r}(U + \varepsilon) = \frac{2}{3 \times 10^{10} \cdot r}(U + \varepsilon) \cdot 10^{14} \qquad (I)$$

$$B_i = \frac{(U + \varepsilon)}{1.5r} 10^4 \text{ gauss}$$

Conditions for magnetic guide field:
Equality of centrifugal force and magnetic force gives

$$e\, v\, B_s = \frac{m\, v^2}{r} \quad \text{or} \quad B_s r = \frac{mv}{e} = \frac{m}{e} \frac{c\sqrt{U^2 + 2\varepsilon U}}{(U + \varepsilon)}$$

$$B_s r = \frac{m_o c^2}{\frac{m_o}{\varepsilon}} \cdot \frac{(U + \varepsilon)}{\varepsilon} + \frac{c\sqrt{U^2 + 2\varepsilon U}}{(U + \varepsilon)} = \frac{1}{c}\sqrt{U^2 + 2\varepsilon U} \qquad (J)$$

From equation (G)

$$\left(\frac{c\, r\, B_i}{2}\right)^2 = U^2 + 2\varepsilon U$$

$$\text{or} \qquad B_i r = \frac{2}{c}\sqrt{U^2 + 2\varepsilon U} \qquad (K)$$

Comparing equations (J) and (K)

$$B_i = 2 B_s \qquad (L)$$

or the field strength of the guide field must be half the average intensity of the total flux within the orbit.

The above basic formulas, especially (I) and (L) are sufficient for the preliminary design of the magnetic yoke and the guide and induction poles of the betatron.

2. Dr. Wideröe's Design Calculations for the 15 Mv Betatron. (Started Jan. 1, 1944)

We shall first compute the flux and induction for a non-saturated betatron. The average induced flux from (L) and (I) is

$$B_{av}\ (15\ \text{Mv}.14\ \text{am}) = 2\ .B_{14\ cm} = \frac{15 + 0.51}{1.5\text{x}14}$$

$$B_{av} = 7400\ \text{gauss}$$
$$B_{14} = 3700\ \text{"}$$

From electrolytic trough measurements the maximum induction in air is indicated to be

$$B_{Ind\ max.} = 3700 \times \frac{0.2555}{0.084} = 11{,}300\ \text{gauss.}$$

Induction flux Φ ind $= \pi\ .14^2\ .7400 = 4.55 \times 10^6$ maxwells.

Fraction of flux carried by the induction pole

$$\Phi\ \text{ind. pole} = \frac{117}{117 + 46.7} \times \bar{\varnothing}\ \text{ind.} = 3.25 \times 10^6\ \text{maxwells.}$$

In the last expression the quantity 117 gives a measure of the area within the induction pole periphery ($\pi\ .9^2$) multiplied by the corrected prevailing field strength (11,300 gauss corrected for the distribution at the edge of the pole).

The figure 46.7 represents the area between the inner edge of the guide pole and the equilibrium orbit multiplied by the corrected average field strength for this region. Correction is made from a curve which gives the prevailing induction as a function of radius as determined by electrolytic trough measurements or flux plots.

The flux in the guide pole is given by:

$$\bar{\varnothing}_s = \frac{46.7 + 109 + 4}{117 + 46.7} \times 4.55 \times 10^6$$

The numerator contains the figures 109 and 4 which are proportional to the corrected flux in the region between the equilibrium orbit and the outer periphery of the guide pole and an additional estimated leakage flux. This gives two fractions of $\overline{\not{Z}}$ which add up to more than unity which is as it should be since the induction $\overline{\not{Z}}$, is definitely only that part within the equilibrium orbit. Addition of the above parts gives for the total flux to be carried by the yoke

$$\Sigma \, \phi = 7.7 \times 10^6 \text{ maxwells}$$

The stacking factor of the radially laminated core is deduced from an easily derived formula $f_e = 1 - \frac{1}{n + 1}$ where n is the number of laminations used per sector. Wideroe assumes nine in the first design giving $f_e = 0.90$ which, corrected for increase of thickness due to varnishing is 0.81.

Estimate of Magnetizing Turns Required:

The ampere turns required for the air gap of 1.96 cm is given by

$$NI = \frac{B \cdot 1}{0.4\pi} = 0.8 \times 3700 \times 6 = 17800 \text{ amp turns.}$$

For the additional air gap as well as for the iron path itself (disregarding saturation) a figure of 800 to 1200 ampere turns may be assumed in addition. The total number of ampere turns thus becomes 18,600 to 19,000.

For a winding of 280 turns the current will be $i_{max} = 66.5$ to 68 amp or $i_{eff} = 47 - 48$ amp. The volts per turn of this winding are

$$U = 4.44 \, \underline{f} \, \overline{\phi}_{max.} \, 10^{-8} = 4.44 \times 50 \times 7.7 \times 10^6 \times 10^{-8}$$
$$= 17.1 \text{ volts / turn.}$$

$U_{280} = 280 \times 17.1 = 4780$ V.

Exciting power or dead load:

KVA = 48 x 4780 = 230 KVA

In the equipment planned there will be 20,700 amp turns or, for a 280 turns winding, about 52.4 effective amperes. The maximum induction flux is 7.4×10^6 maxwells and the voltage 4600 for KVA = 242 KVA. The size of capacitors is 11 microfarads. Capacitor reactance $X_c = \frac{1}{wc} = \frac{1}{314 \times 11.10^{-6}} =$ 289.5 ohms.

For 4780 v and 3 capacitors in parallel

$$I = \frac{4780}{96.5} = 48.5 \text{ amp.}$$

<u>Computation of the winding:</u>

Assume 4 rectangular wires of 3.5 x 3.5 mm cross section = 49 mm^2 and a current density of 1 Amp/mm^2. We shall wind each 140 turn double coil as four individual coils connected in series each containing 35 layers. Average length of winding $l = 2\pi \times 24.2 = 154$ cm.

Coil resistance $R = \frac{0.018 \times 1.84}{49} = 0.079$ ohms per coil.

Losses: $I^2R = 2410 \times 0.079 = 190$ W. Considering additional (eddy current) losses we may assume about 250 watts per coil.

Cooling Surface

$$F = 2 \times (14 \times 154) + 2\pi \times 31.2 \times 8.6 = 4300 + 1690$$

The product 14 x 154 is an approximation for:

$$\pi (31.2^2 - 17.2)^2 = 4250 = 5990 \text{ cm}^2.$$

Thus we have $\frac{250}{5990} = 0.0418$ watt/cm^2 which should give about a 42° C temperature rise.

Weight per Coil.

$$G = 8.9 \times 140 \times 4 \times 15.4 \times 12.3.10^{-4} \text{ kg} = 94 \text{ Kg.}$$

A "coil" is a double coil having 140 turns total.

Expansion of Equilibrium orbit by Saturation.

Conditions $\frac{\int_0^{r} 2\pi r B \, dr}{\pi r^2} = B_r = 2 B_s$

We shall increase a fraction of the induction flux to (1 + J) times that of the remaining flux and obtain a radial expansion r_o (1 + Δ) inserting these conditions in the 1:2 rule:

$$\frac{V_o 2.\ 2B_o \left[d\,(1 + \Delta) + (1 - d) \right] + 2 \int^{n_o + \Delta} B\, r.dr}{r_o^2\, (1 + \Delta)^2} = 2\, B_o\, (1 - K\, \Delta)$$

Now $B_r = B_o \left(\frac{r_o}{r}\right)^K \sim B_o\, (1 - K\, \Delta)$

So

$$\int^{r_o + \Delta}_{r_o} B_o\, r \left(\frac{r_o}{r}\right)^k dr = B_o r_o^k \left. \frac{r^{2-K}}{2-K} \right|_{r_o}^{r_o(1 + \Delta)} = B_o\, r_o^2\, \Delta$$

$$\left[d(1 + \delta) + (1 - d) \right] + \Delta = (r K \Delta) \quad (1 + \Delta)^2 \quad \Delta = \frac{d\, J}{1 - k}$$

Example: δ = 10, 12, 14, 16, 18, 20%
r_o = 16, 5, 17, 17.5 18, 18.5, 19

The flux conditions for r = 18 cm (expanded orbit) shall now be computed for an energy of 15 Mv. Assuming that the flux in the center pole is increased by δ, = 11% and that the flux in the guide pole is decreased by δ = 8%, we obtain:

$$B_{av.18\ cm} = \frac{15.51}{1.5 \times 18} = \underline{5750\ g.}$$

$$B_{ind.18\ cm} = \pi \times 18^2 \times 5750 = \underline{5.85 \times 10^6\ max.}$$

$$\underset{(10\ cm)}{\phi\ (cent,\ pole)} = \frac{117 \times 1.11}{117 \times 1.11 + 96.5 \times 0.92} \times 5.85 \times 10^6 =$$

$\underline{3.48 \times 10^6 \text{ max.}}$

$$\underset{(10\ cm)}{\Phi\ (guide\ pole)} = \frac{159.7 \times 0.92}{117 \times 1.11 + 96.5 \times 0.92} \times 5.88 \times 10^6 =$$

$= \underline{3.92 \times 10^6 \text{ max.}}$

$$\Sigma\, \Phi = 7.4 \times 10^6 \text{ max.}$$

There is a reduction of 7.7 - 7.4 = 0.3 x 10^6 or 4% of the total induction flux compared to the non saturated state. The ampere turns required will increase about 13%.

To diminish the induction in the core its diameter is increased to 22 cm.

$$B = \frac{3.48 \times 10^6}{\pi 9^2 \times 0.81} = 16900 \text{ g.}$$

Amp. turn 7.4 x 120 = 890 A.T.

Maximum induction for the remaining portion of the center pole

$$B_{max} = \frac{3.48 . 10^6 + 0.67 . 10^6}{\pi \; 11^2 \times 0.81} = 13500$$

Minimum induction for this part

$$B_{min} = \frac{3.48 \times 10^6 + 0.242 \times 10^6}{\pi \, 11^2 \times 0.81} = 12,200 \text{ g.}$$

Computing on the basis of an average inducing flux of 12800 g. we have:

Amp. turns = 22 x 11 = 242

Av. induction in air for $r = 10$ cm.

$$B_{av} = \frac{3.48 \; . \; 10^6}{\pi \; 10^2} = 11,100 \text{ g.}$$

To determine the maximum (determining also the ampere turns required) induction, use is made of the same relation used in the unsaturated state

$$B_{max}/B_{av} = 11300/10350 = 1.09$$

Maximum induction in air = 11,100 x 1.09 = 12,100.

Amp turns (air) = 0.8 x 1.96 x 12100 = 19000 amp. turns.

Total amp. turns for the core (center pole) ΣA.T. = 20,130 A.T.

The iron of the guide pole now requires dimensioning so that the same number of ampere turns are obtained with the prescribed flux.

$$\text{Amp. turns (air)} = 19000 \times \frac{0.92}{1.11} = 15{,}750 \text{ Amp. turns.}$$

$$\text{Check: Amp. turns (air)} = 17800 \times \frac{3.92}{4.44} = 15{,}750 \text{ in agreement.}$$

For the iron path the amp. turns = 4380, i.e. $\theta\ F_e = 200$ amp. turns/cm. (Iron path = 22 cm).

Approximating the magnetization curve by a straight line we have:

$$B = 13800 + 1.25 \times 20 \times \theta \text{ cm/A.T.}$$

and we compute the stray flux assuming: Iron surface = 180 cm^2 for the stray flux. We have for the average of the effective amp. turns.

$$\text{Amp. turns} = \frac{4380 - 120}{2} = 2130 \text{ Amp.turns per core.}$$

$$z = \frac{1}{\mu \times 1.25 \times 180 + 1.25\ (\pi(+ 8.5^2 - 13.8^2) - 180)}$$

$$\frac{\text{amp.t.}}{\text{max}} = \frac{1}{4500 + 370}$$

$$\mu = 20$$

$$= \frac{1}{4780} \quad \frac{\text{att.}}{\text{max}}$$

$$Y = \frac{2\ \pi\ \times 12.25}{2.8} \times 1.25 \quad \frac{\text{Max wr}}{\text{A.t.}} = 34.4$$

$$\alpha = 0.0839 \sqrt{\frac{Y}{Z}} = 408$$

$$\text{for. } r = 11 \text{ cm.} \quad da = 2.5 \quad \phi_o = 0.827 \times 10^6 \text{ max w.}$$

$$\phi_m = 1.20 \times 10^6 \text{ max w.}$$

For the stray flux passing directly to the pole surfaces we assume the following approximate values. The magnetic conductivity is

$$\underline{\lambda} = \frac{2\pi\ \times 11 \times 3.3}{2.0} \times 1.25 \quad \frac{\text{Max w.}}{\text{A.t.}} = 142. \text{ Max w.}$$

$$\phi_s = 2070 \times 142 = 0.292 \times 10^6 \text{ max w.}$$

$$\Sigma\ \phi_s = 1{,}492 \times 10^6 \text{ max w.}$$

The purely ferritic flux, is

$$\frac{\phi}{E_e} = 3.92 \;.\; 10^6 - 1.492 \;.\; 10^6 = 2.43 \;.\; 10^6 \text{ max w.}$$

This indicates a ferritic induction of 2.43/180 = 13500 g. in very good agreement with the assumed value of 13.800 g. The greatest uncertainty is in the direct stray flux ϕ_{s_1}, and it is useless to attempt to estimate it exactly.

For the width of the iron we have

$$X = \frac{180}{2\ \pi\ .\ 13.8 \times 0.9} = 2.30 \text{ cm.}$$

An induction of 10,200 is chosen for the yoke. The section required is:

$$F = \frac{7.7 \times 10^6}{10.2\ .\ 10^3\ .\ 0.9} = 838 \text{ cm}^2 = 2 \times 32.2 \times 13.0 \text{ cm}^2 \quad \text{(width) (height)}$$

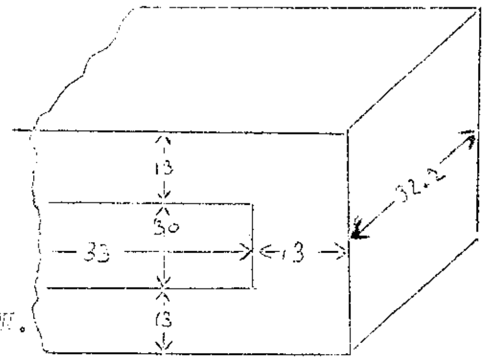
Av. yoke length = 79 + 43 = 122.

Necessary Amp. t = (3 x 133) = 400 A.t.

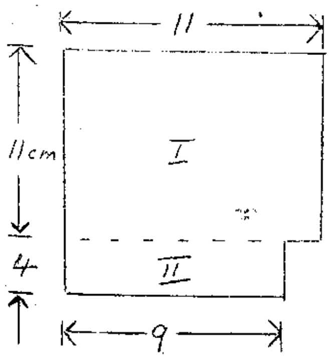
Total weight G = 76 x 0.9 x 837 cm^2 . 124

= 710 kilog.

Total losses W = 1.3 x 1.04 x 710 = 960 W.



Center of induction pole.



Changed to u c m 9/3/44.

Stacking Factor

I. Weight = 7.6 x 0.9 x 0.9 π 11^2.11 = 25.7 kg.

I. Losses = 25.7 x 1.3 x (1.28)^2 = 55 W.

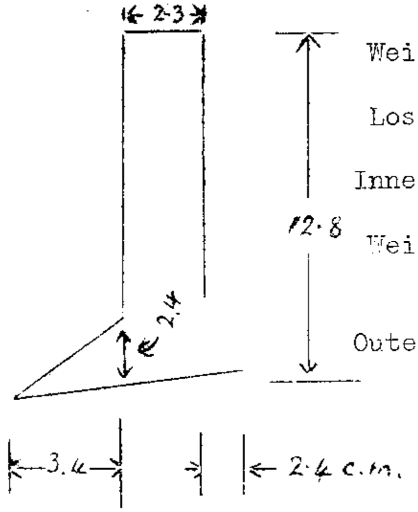
II. Weight = 7.0 x 0.9 x 0.9 π 9^2.4 = 6.3 kg.

II. Losses = 0.3 x 1.3 x (1.69)^2

Σ G = 32 kg $\quad$ Σ loss = 78 W.

Center strip. (2 π x 14.0 x 2.3 x 0.90)

Guide Pole



Weight = 7.6 x 180 x 12.8 = 17.5 kg.

Loss = 17.5 x 1.3 x $(1.78)^2$ = 72 watts.

Inner pole shoe.

Weight = 7.6 x 2 π x 10.4 x 0.9 x 1.2 x 3.4 = 1.8 kg.

Outer 7.6 x 2 π x 13.8 x 0.9 x 1 x 2.4 = 1.4 kg.

Losses estimated at 3 watts/kg.

Total weight 20.7 kg. Total losses 75 watts.

Summary of weight and losses for whole machine.

	Weight	Losses
Yoke	710 kg.	960
Two Induction poles	64	156
Two Guide poles	42	150
Iron	816	1266 watts
Two copper coils	188	500 "
Total	1004	1700 w.

Volts per turn for r = 14 cm.

$= 2 \pi f \times \bar{\emptyset}_m 10^{-8} = 314 \times 7400 \times \pi \times 14^2 \times 10^{-8} = 14.3$ V.

For r = 17 cm.

$= 14.3 + 314.\ 10^{-8} \times \pi\ (17^2 - 14^2)\ 3700 \times 0.9^4 = 14.3 + 3.22 = 17.52$ V.

C. Projected Designs for 100 Mv and 200 Mv Betatrons.

1. The 100 Mv. Betatron with Air Gap.

In passing to a description of the 200 Mv. design some published notes on (1) on the 100 Mv intermediate design may be reviewed. The basic considerations on the construction of very large betatrons depend largely on experience gained in the construction of cyclotrons and in the construction of very large transformers. To keep the power requirements down to a minimum the air gaps of the machine must be kept to the smallest figure by the dimensions of the acceleration chamber.

(1) cf. Wideroe, Archiv F. Elektrotechnik 37 (1943) p. 391-408.

For a given electron injection voltage the radiation load increases approximately as the square of the air gap while the power, together with iron and copper losses, increases in approximately direct proportion.

To obtain a most economical design the transformer construction developed by the Brown Boveri Co. may be adapted. This has a radial construction in which the radial laminations are welded at the top and bottom. In this type of construction an exactly circularly-formed iron section is obtained and the required guide-pole shapes are easily designed and adapted. The necessary cooling ducts are likewise easily incorporated into the core.

The minimum amount of iron is used if the yoke is continuous as in a shell transformer. This lends itself to a rugged construction having a small yoke height and large cooling surface. It also offers a certain amount of radiation protection by shielding the acceleration tube, access to which it obtained by apertures cut into the yoke.

A preliminary design for a 100 Mv unit is shown in Figure 4. Here yoke, core, and iron return form a common iron unit constructed from radial laminations. By removal of the 10 ton upper half, access is had to the windings and acceleration tube. The core is crossed with air ducts to remove the 13.6 kw iron loss (0.35 mm laminations with loss of 1.1 w/kg). The maximum induction of the core is 14,500 gauss for a stacking factor of 77%. The yoke and iron returns are able to dissipate 28 K.W. by surface cooling. In these parts the stacking factor is taken as 90% and the maximum inductions chosen are 14,500 and 12,500 gauss respectively. The total weight is 20 tons.

The air gap for an equilibrium orbit of radius = 60 cm is 5 cm and about 20,000 effective ampere turns are required to produce in it a guide field of 5,600 gauss. To accomodate the exciting winding (46.5 kg of copper, assuming a current density of 2 amps/mm^2) a window of 800 cm^2 cross section is provided. Thus a space factor of 12.4% is obtained, allowing insulation of the winding to 30-50 K.V. This voltage is so chosen that it will be the most economical one, considering cost of capacitors. The copper losses of the transformer will be 5-6 K.W. and the required dead load 7500 KVA. The purely constructional costs aside from development and research is estimated at 80,000 RP_x for the betatron, 120,000 RM for the capacitors and the possible total cost is about 200,000 RM, comparable with that for an 8-12 Mv (deuteron energy) cycletron.

(a) Improvement of Yield by Means of Auxillary Lenses.

It is the hope of Wideroe and his group to increase the electron filling by means of electrostatic or electromagnetic lenses. In the 100 Mv unit it was planned to fill the air gap as completely as possible with the acceleration chamber and to provide multiple injectors and guide fields on its periphery. Allowing four guide fields for each oscillation of the electron, 28 cathodes will be required for 7 oscillations per revolution. With an average electron beam of 1 ma (V_i equals 300 kv) the frequency of the injection voltage will be 17,000 cycles for a 100 Mv betatron operating at 50 cycles. This gives a charging period of 2 x 10^{-5} seconds per cycle. The average charging current for these times must therefore be 1 ampere and requires, assuming an efficiency of 10%, a total cathode emission surface of 500 mm^2. For 28 cathodes this amounts to 18 mm^2 per cathode. Cathode windows will also have to be of this order of size. To prevent any electronic charges building up on the acceleration tube the outside should be neutralized and grounded.

(b) General Estimate of Weight, Size and Cost.

X-rays may be produced by saturating either the induction or the guide fields. Increasing the frequency of operation rapidly increases the power demands. A review of the power and cost of large accelerators has been summarized by Wideroe in the following table.

Maximum Voltage U_{max}	100	200	400	600	800	1000
Acc. tube radius	0.6	1.2	2.4	3.6	4.8	6.0m
Injection voltage	300	300	300	300	300	300 kv
Av. electron current 50 cycles	1	2	4	6	8	10 ma
Av.radiation load	100	400	1600	3600	6400	10,000 kw
Core losses	48	400	3200	10800	25600	50,000 kw
Efficiency	67	50	33	25	20	16.7 %
Magnetization load, Mvars	7.5	30	120	270	470	750 Mvars
Core height	1.02m	2.04	4.08	6.12	8.16	10.2 meters
Core diameter	2.44	4.88	9.76	14.64	19.52	24.40 m
Core weight	20	160	1780	4320	10240	20,000 tons
Copper weight	0.5m	4	32	108	256	500 tons
Est.cost Betatron	0.08	0.64	5.1	17.3	41.0	8×10^6 RM
Est.cost capacitors	0.12	0.48	1.9	4.3	7.7	12×10^6 RM

All of these figures are based, of course, on the above described 100 Mv design.

2. Projected Design of 200 Mv Betatron.

No attempts were made to go on with this after it became known in 1943 that a 100 Mv unit had been completed and set into operation in the U.S. It was then felt that nothing less than a 200 Mv machine would be worth a project and attention was also turned to a new design in which the air gap of the machine was eliminated.

Even with this feature the diameter of the machine would be about 3 meters. Since Dr. Wideroe has close connections with the firm of Brown Boveri who furthermore are skilled in the construction of large transformers, this firm was engaged for the design and construction but not for the installation. For the latter phase an independent contract was planned to take effect on the completion of the machine. Construction work, according to Dr. Wideroe's plans and computations, was begun in August 1944. In mid-October a first crude design had been made which was discussed at length in Heidelberg. In this conference the M.V.A. was represented by Dr. Wideroe and Dr. Kollath and the Brown Boveri firm by its Director, Dr. Meyer-Delius, Chief Construction Engineer Otto Weiss, and Dr. Helmut Boecker, detail designer. The construction of models and a small experimental unit for study of special problems was also considered at this time. Exact calculations were to be completed before the end of the year. In November, M.V.A. received some questions and designs to which reply was made by letter. In January, the first plans for construction were complete and the M.V.A. was informed, but it was too late for another conference and discussion of these plans. Since that time, the M.V.A. group has had no contact with B.B.Co. and all work on this project came to an end with the occupation of Germany early in 1944.

(a) General Description of the 200 Mv unit.

The conference held at Heidelberg evolved a design which had the following general characteristics. Referring to Figures 5 and 6. A radially laminated iron core of 1320 mm. diameter was to be surrounded by ring-shaped radially laminated parts of about 200 mm. width. These carry the guide poles which have an average separation of 7 cm. and also the compensating windings. About these are placed the main exciter windings divided above and below the median plane. To complete the magnetic induction circuit four mantels 90 degrees apart were provided on the circumference. A difficulty arising from this feature will be discussed below. The preliminary problems of the original design included the purely mechanical construction of the laminated parts of the machine, the fastening of the guide poles to the machine, the division of the transformer to facilitate installation of the acceleration chamber, pumping, and electrical connections, and adjustments to guide poles. It was planned to purchase about 40 tons of 0.35 mm. transformer sheet (29 gauge) having a core loss of about 1.1 w/kilogram. This material had a price of about 80 pfennigs per kg.

The upper pole shoes were to be capable of being raised, and the four mantel yokes were to be displaced radially to allow access to the acceleration chamber. The spaces between the mantels give access for leads, etc. A special difficulty was in the evacuation and cooling of the exciting windings and it was proposed to mount the inner winding directly adjacent to the core in a stationary oil case.

(b) Final Form Decided Upon.

The general features of the final form evolved in the above design work are shown in Figure 6. In vertical section is shown the central core with its cooling system consisting of a number of parallel helically shaped cooling ducts winding through the body of the core and connecting at the upper and lower ends of the core to the compressed air inflow and return ducts 5. The core has a stacking factor of about 70%. By means of ledges about its periphery a ring of radially laminated pieces making up the pole shoes are held in proper position, and so that the upper pole shoe ring may be raised together with the connecting ring 3. This carries on the lower side the main A.C. exciter winding and the D.C. premagnetizing winding. The original intention to complete the gap-less magnetic circuit by means of the four mantels led to the difficulty that the uniformity of the magnetic flux in the guide poles will be disturbed by the locking of the mantels to the guide poles at only a few points. To avoid this difficulty, it was proposed to add an additional ring 3 (horizontally laminated) which serves the purpose of smoothing the flux from the core into the mantels 4. This is broken between adjacent mantels with a zig-zag joint to avoid an electrically continuous circuit about the flux passing into the guide poles. The figure shows the principal and compensating magnetizing windings 8 and 9, and the direct current premagnetizing windings 10 and 11.

By eliminating the air gap in this design a high central accelerating flux is produced by the main windings. These windings also serve to produce the guide field which at all times has to bear a fixed relation to the induction flux to maintain a stable electron orbit. In this design it is furthermore proposed to premagnetize the guide field so that it will be operative over a half a cycle instead of a quarter cycle, thus permitting the induction field to accelerate the electrons for more than a quarter period. This principle is illustrated in the accompanying figure, Figure 8.

The air gap of the earlier design provided the necessary relationship between the induction and guide fields by insuring their proportionality. Since the use of it leads to unnecessarily large power requirements an attempt is made to provide the necessary relationship by a winding on the inner coil space (compensating winding) on which is impressed a fixed potential of the corresponding magnitude. For practical purposes the main magnetization winding and its compensating windings may be connected to the same driving source and the flux vs. induction relationship may be regulated by proper proportioning of the windings of the two coils. For fine adjustment a regulation transformer may also be connected into the circuit. Figure 7 shows the manner in which these windings are to be energized from power sources and the necessary compensating transformer connection to neutralize the A.C. voltage induced in the D.C. winding.

For this design the following windings were planned: (See Figure 5)

1. An alternating current winding, placed outside of the guide pole.

2. A compensating winding correspondingly placed about the induction pole to depress the A.C. magnetization of the core to such an extent that the average induction of the core is only twice that of the induction at the center of the guide pole (i.e. fulfillment of necessary 1:2 relation).

3. A premagnetization winding for the guide pole of which the induction is to vary from 0 to a maximum.

4. A compensating winding corresponding to the above which while in principle is not necessary is used to prevent an undue loading of the machine by a D.C. magnetization with undesirably high induction and saturation characteristics.

Further general electrical characteristics on the design are:

Total induced flux	105×10^6 maxwells
Induction in core	15,000 gauss
yoke	13,900 "
mantels (4)	12,200 "
guide pole	15,000 "
Total iron weight	26 tons
Volts/turn at equilibrium orbit	366 volts
Max. energy	200.5 Mv
Operating power driving circuit	7,900 kva
Operating power compensating windings	4,800 kva
Resulting net load	3,100 kva
Iron losses	about 67 kw

The details on exact form of the guide poles were not determined, there being a feature to be worked out during the construction.

(c) Stablization of Orbit and Improvement of Orbit Control.

The magnetic stabilization forces, brought about by the suitable shaping of the guide poles, are known to diminish with an increase in size of the betatron while retaining the same sectional dimensions of the acceleration tube. Wideroe and Touschek have made a theoretical study of the additional stabilization that may be achieved by electron optical lenses. These may, in their simplest form, consist of a set of series connected coils spaced closely around the circumference of the acceleration chamber, the axis of each being a tangent to the orbit. The principal advantage of such a lens system is that a greater electron filling of the tube should be possible.

The maximum beam current is, in theory, determined by the effect of the mutual repulsion due to Coulomb forces of the electrons. These forces are compensated again by the magnetic forces mentioned above. Excess electrons thus injected are driven out of the beam and end on the walls of the acceleration chamber. The repulsive forces decrease with time after injection of the electron and are greatest at the instant of injection while the electrons are still relatively slow. The effectiveness of the electron lens will be greatest in the early stages of the injection process. Wideroe has estimated that in the 200 MV design it is possible to increase the stabilization forces for 10 kV injection about 1350 fold and that this in turn would increase the circulating current from 0.1 to 135.0 microamperes. In the 15 Mv betatron the stabilization forces are greater and the relative improvement to be expected is much less. The improvement factor is about 45 which would increase the current from 0.5 to 22.5 microamperes.

It was learned on a visit to MVA that the magnetic lens system had been tried on the 15 MV betatron without any marked success. It was, however, not regarded as an abandoned project but was included in the experimental research work to be followed.

A further subject of concern in the design of the 200 MV machine was the possibility of the energy losses in the machine by radiation damping announced by Iwanenko and Pomerantschuk (Phys. Rev. 65 p. 43). The consequence of their theory were discussed by Dr. Touschek and Dr. Lenz or Hamburg. Their view is that as a result of radiation damping a radiation with a frequency of the order of 10^7 to 10^8 cycles is generated. This radiation behaving like visible radiation, may be reflected by the metallized walls of the acceleration chamber. This would fortunately hinder the passage of the damping radiation out of the machine as in a wave guide, and eliminate the fear that this would be a decisive factor in the operation of very large machine. Certainty on this point however, did not prevail and it was hoped to study such efforts experimentally on a smaller machine before completion of the large machine. It was believed that a possible remedy lay in increasing the driving frequency considerably.

(d) Preliminary design calculation of Dr. Wideroe for 200 MV Betatron:

Equilibrium orbit radius $R = 70$ cm.

For premagnetization $\bar{B} = \left(\frac{u + 0.51}{3R}\right) 10^4$

$$\bar{B} = \left(\frac{200 + 0 \cdot 51}{210}\right) \times 10^4 = 9550 \text{ gauss}$$

$$\Sigma \phi = 70^2 \,.(\pi).9550 = 147 \times 10^6 \text{ maxwells}$$

$$= \pi\,(70^2 - 60^2)\,\frac{\bar{B}}{2} + \xi_k \,.(\pi).60^2.Bu$$

$$= 19.5 \times 10^6 \text{ max.} + \xi_k \times 11300 \text{ Bu}$$

in which ξ_k is the iron stacking factor = 0.75

$$B_\mu = \frac{(147 - 19.5)\ 10^6}{0.75 \times 11{,}300} = 15{,}000 \text{ gauss}$$

Calculation of the yoke height (Induction = 12,500 g)

$$\xi . \pi . R_k^2 \,.\, Bu = \pi R_u h \,.\, \xi_2 .\, Bg \quad .\ \xi = .90$$

$$\text{or} \quad h = \frac{\xi u.\ Bu}{\xi_2 .\ Bg} \cdot \frac{Ru}{2} = \frac{0.75 \times 15000}{0.9 \times 12500} \times \frac{60}{2} = 30 \text{ cm.}$$

Yoke flux for R = 74 cm :-

$$\phi = 147.10^6 + \frac{9550}{2}\ \pi\,(78^2 - 70^2) = (147 + 18)10^6 = 165 \times 10^6 \text{ max.}$$

$$B_2 = \frac{165 \times 10^6}{0.9 \times 2\pi \times 74 \times 30} = 13{,}100 \text{ gauss}$$

An application of C. H. F. Müller A 3224 of 25/9/44 describes an electron injector in which the equilibrium orbit is allowed to pass without obstruction through the injection structure. The application also includes the use of a manifold of such injector units about the accelerator orbit.

A very late application C. H. F. Müller, Dr. Mü/Rö/160 Jan. 26, 1945 described an iron-less betatron. The scheme, in which inductions considerably greater than 10,000 gauss are possible, is to consist of a coil of few turns, and an acceleration tube of relatively small dimensions. The coil is of a hemispherical shape to give proper direction to field lines and proper distribution of flux through the orbit of the acceleration chamber. Because of the high power requirements due to lack of iron, this is to be operated by a sudden discharge of electricity stored in a capacitor bank. It is also proposed to include proper proportioning of the induction and guide fields, to divide the winding and endow its parts with different time constants by proper design of self inductance or by inclusion of external circuit inpedance. The application further includes possible use of iron in such parts of the equipment where fields of 10,000 gauss are to be used. Iron, however, is not to be used in regions of most intense fields, that is, 20,000 to 100,000 gauss.

V. Summary and Conclusion.

The principal betatron developments which have been carried out during the war by several German firms and organizations have been discussed in some detail. In the way of evaluation of these developments, especially in comparison with developments which have proceeded simultaneously in the USA, the following notes may be added:

The developments seem to have the same heritage in preceding scientific literature; the main reference sources are the work of Wideroe and the patents of Steenbeck and Kerst, on which developments agreeing fairly closely in general principle have been made. The success of these developments has been of various degrees. The work of Wideroe and the associated groups at MVA and BBC have resulted in an operating 15 MV accelerator which is described above, and some projected designs. The x-ray output of this machine, reported to be as high as 1 kg of radium and as low as 30 g at various times of experimental work, compares fairly well for the upper figure with

American 20 MV machines, allowance being made for the difference in upper energy. The technique of electron injection, while subject to theoretical investigation, lacks considerably in refinement, perhaps partly due to the difficulties encountered in pursuing experimental work at all. The machine of Wideroe presents a somewhat different design from that of Kerst in that saturating guide poles are used. This feature, however, has opened up some design possibilities which could not be foreseen in the Kerst design. In the Kerst equipment, guide fields are produced on the same pole unit carrying the accelerating flux. A further advantage to be emphasized is the weight expansion which in the case of the 15 MV machine is 1200 kg or about 1/2 tons. This is to be compared to a 20 MV machine weighing around 4 tons. On the basis of energy, proportional to size, and weight proportional to the cube of size, a weight of $\frac{20^3}{15^3}$ x 1.5 or about 3.5 tons maximum may be expected. This is a matter however in which the air gap chosen makes a big difference.

The most interesting and remarkable development due to the Wideroe group has been the design of a 200 MV unit in which very considerable economy in iron may be found. An interesting comparison may be made here, in the way of weight, between this design which is to weight about 40 tons for a 200 MV rating and the 130 tons of the American General Electric 100 MV machine. In the 200 MV design certain improvements, such as the use of premagnetization, are considered. It may be premature and optimistic to believe that these ultimate refinements in betatron design will be realized until they are demonstrated on a smaller scale unit, however, they excite admiration for inginuity and good design.

The most successful betatron development observed is perhaps that of Siemens-Reiniger in their 6 MV unit operating at 550 cycles. This unit also has good output for its size and weight and operates with almost unbelievably-steady output. The actual details of electron injection follow American practice to some degree with the difference that inside injection is employed. Some interesting difference exist in the electron starting. X-ray production occurs as in larger American machines, by orbit expansion, but somewhat different orbit expansion and arrangements are used. The most interesting feature, it must be emphasized, is the adaptability of the unit to various arbitrary alterations for the purpose of testing schemes of electron injection and acceleration. American accelerator developments have advanced to the use of sealed-off tubes made of porcelain. The use of porcelain, however, was anticipated by Steenbeck who, in 1943, used a tube of such material.

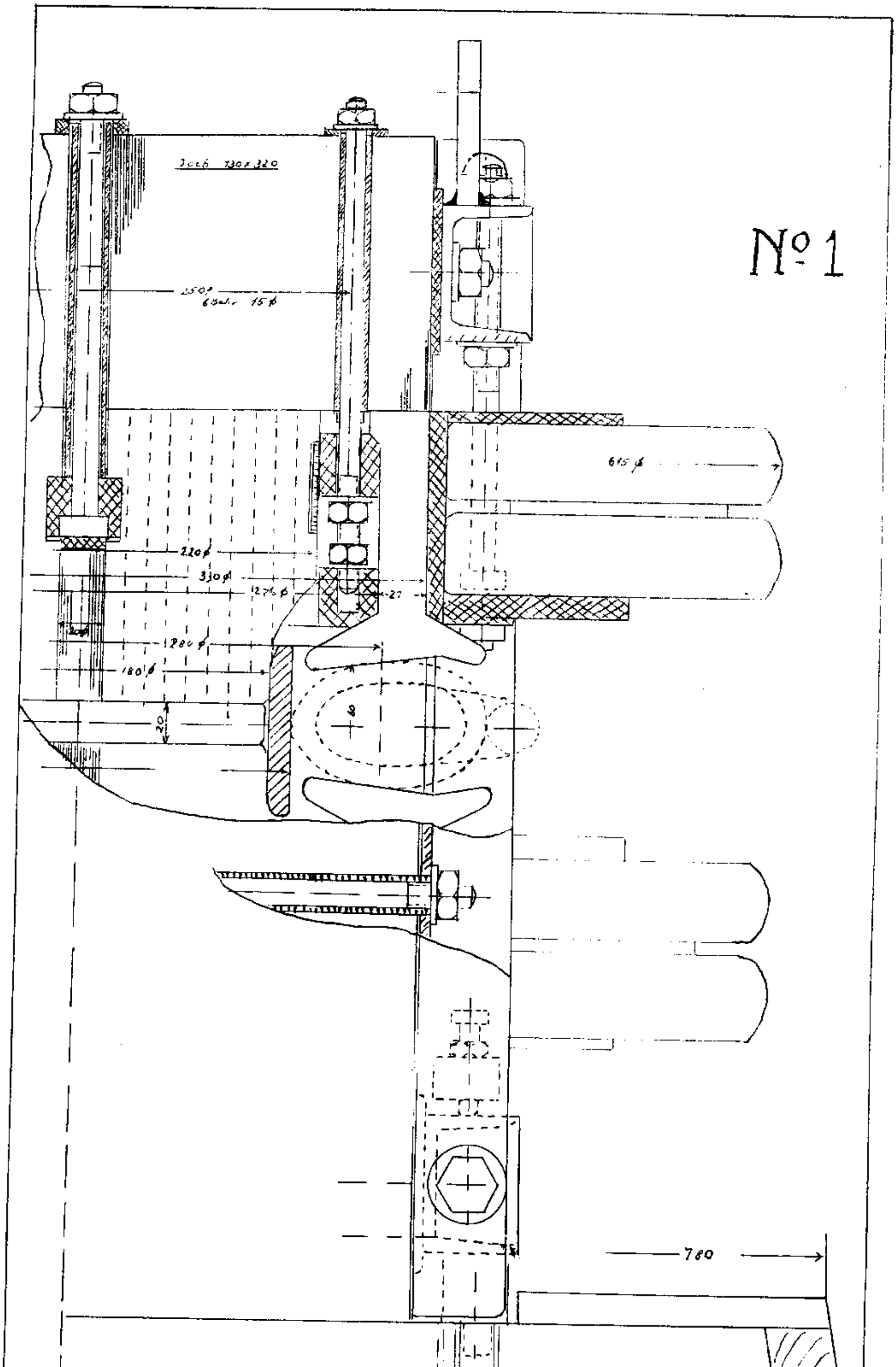
Unfortunately, in this investigation, no visit could be made to Berlin. It is understood that the AEG was also active in the betatron field and that its work was largely based on the proliminary achievements of Wideroe and Steebeck.

Rolf Widröe ? Steen-beck

In connection with the developments reviewed above are to be noted many interesting and ingenious technical details. A review of the patent applications indicates a wealth of ideas which have only in part been systematically studied.

Prepared by:

H. F. Kaiser,
Technician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Joch 730 x 320
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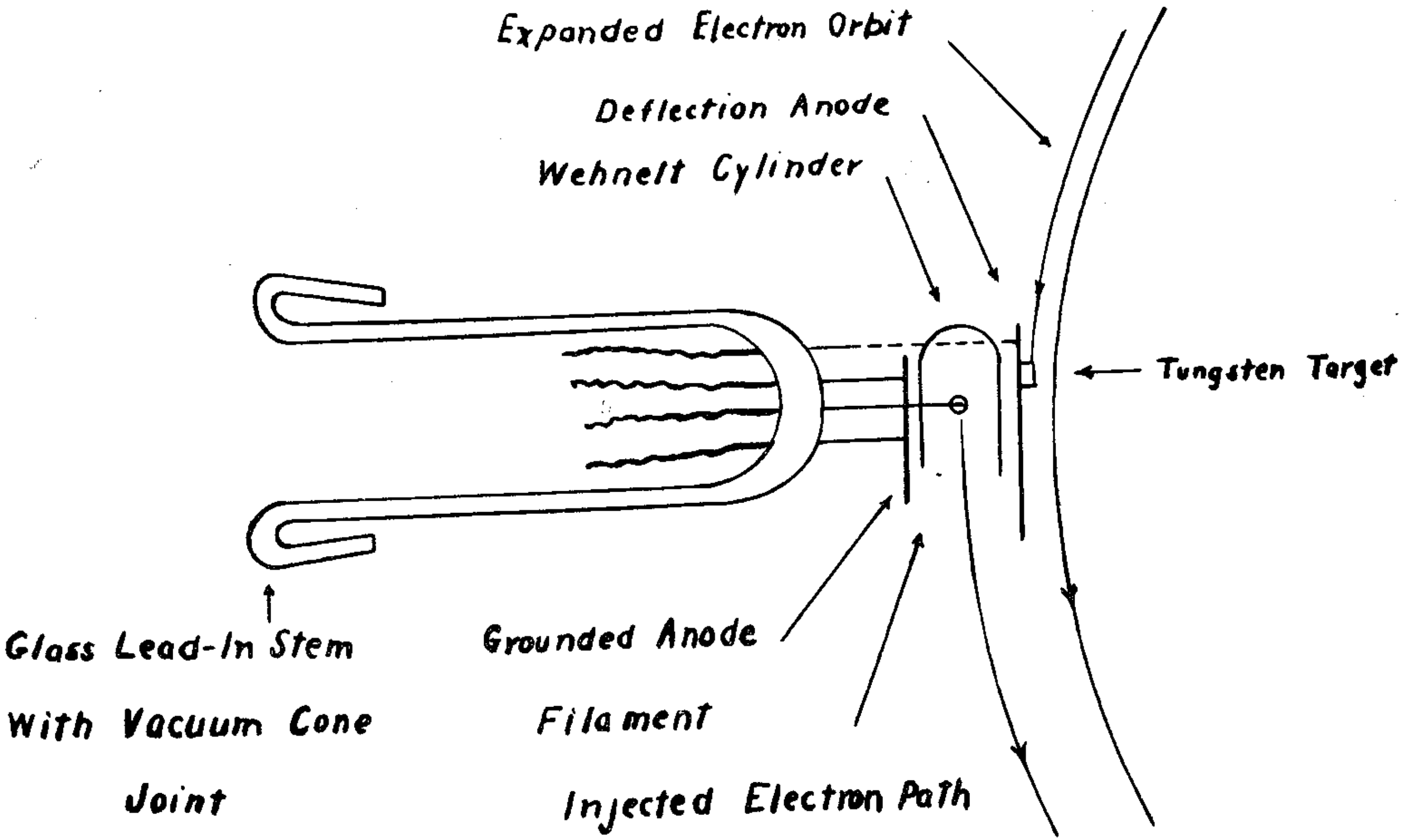
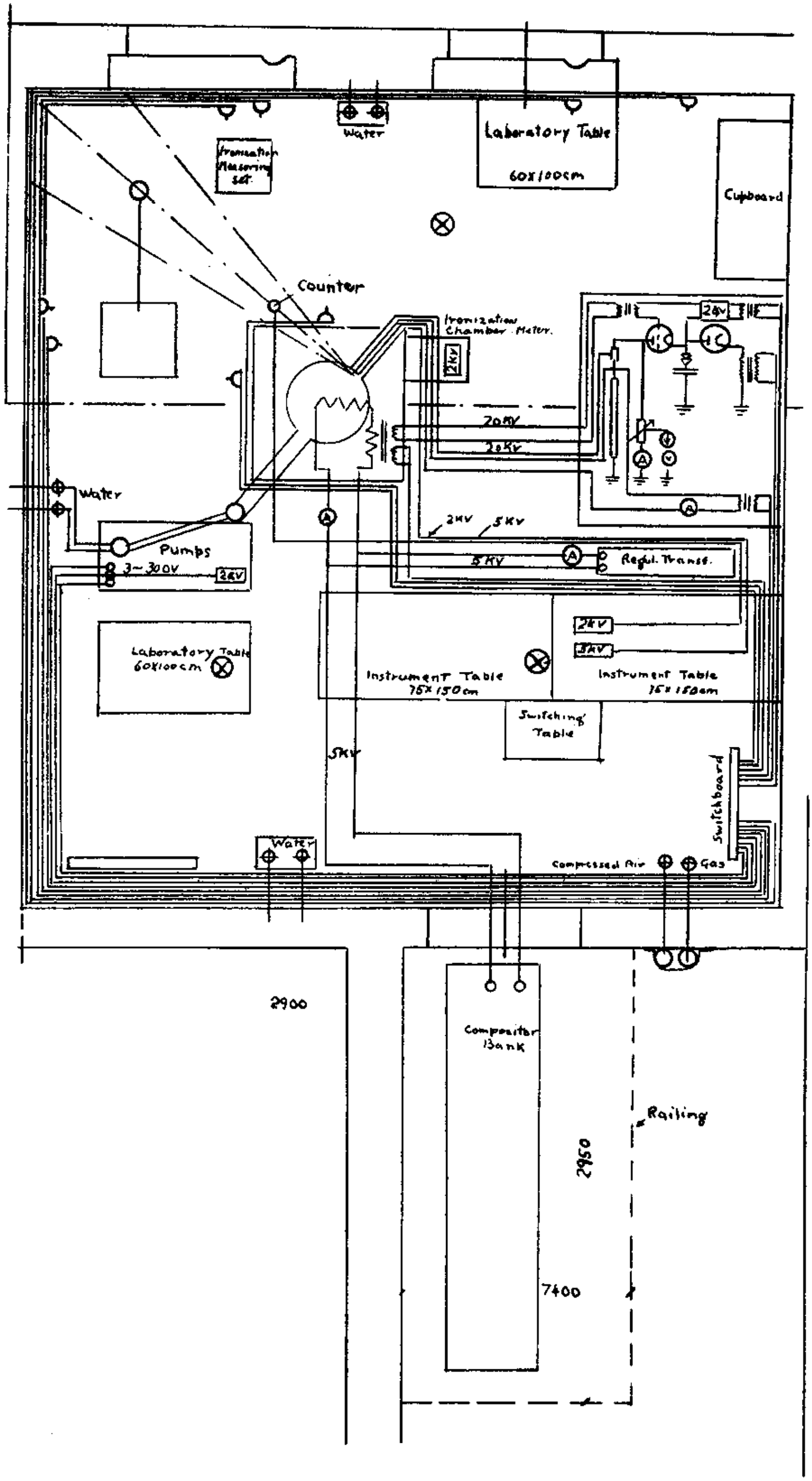

Expanded Electron Orbit
Deflection Anode
Wehnelt Cylinder
Tungsten Target
Glass Lead-In Stem
With Vacuum Cone
Joint
Grounded Anode
Filament
Injected Electron Path

FIG. 2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Laboratory Table
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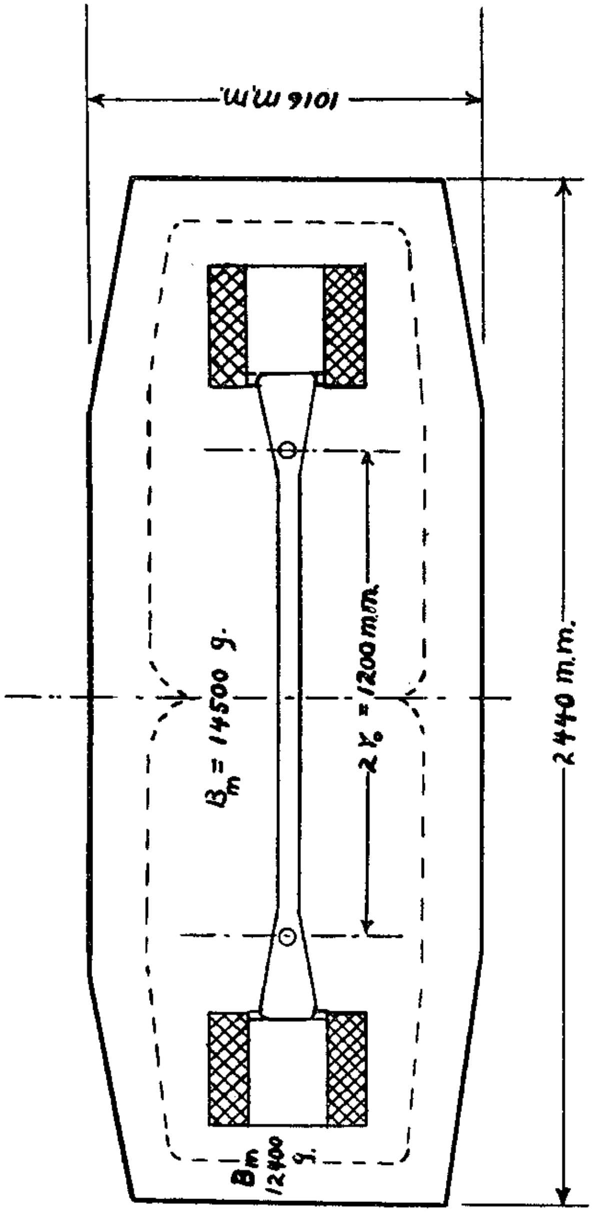
Ionization Measuring set
Cupboard
Counter
Ionization Chamber Meter
2KV
24V
20KV
20KV
Water
2KV
5KV
Pumps
3~300V
24V
5KV
Regul. Transf.
24V
5KV
Laboratory Table
60×100cm
Instrument Table
75×150cm
Instrument Table
75×150cm
Switching Table
5KV
Switchboard
Water
Compressed Air
Gas
2900
Compositor Bank
Railing
2950
7400

Fig 3

FIG. 4



Design for a 100 Mv. Betatron

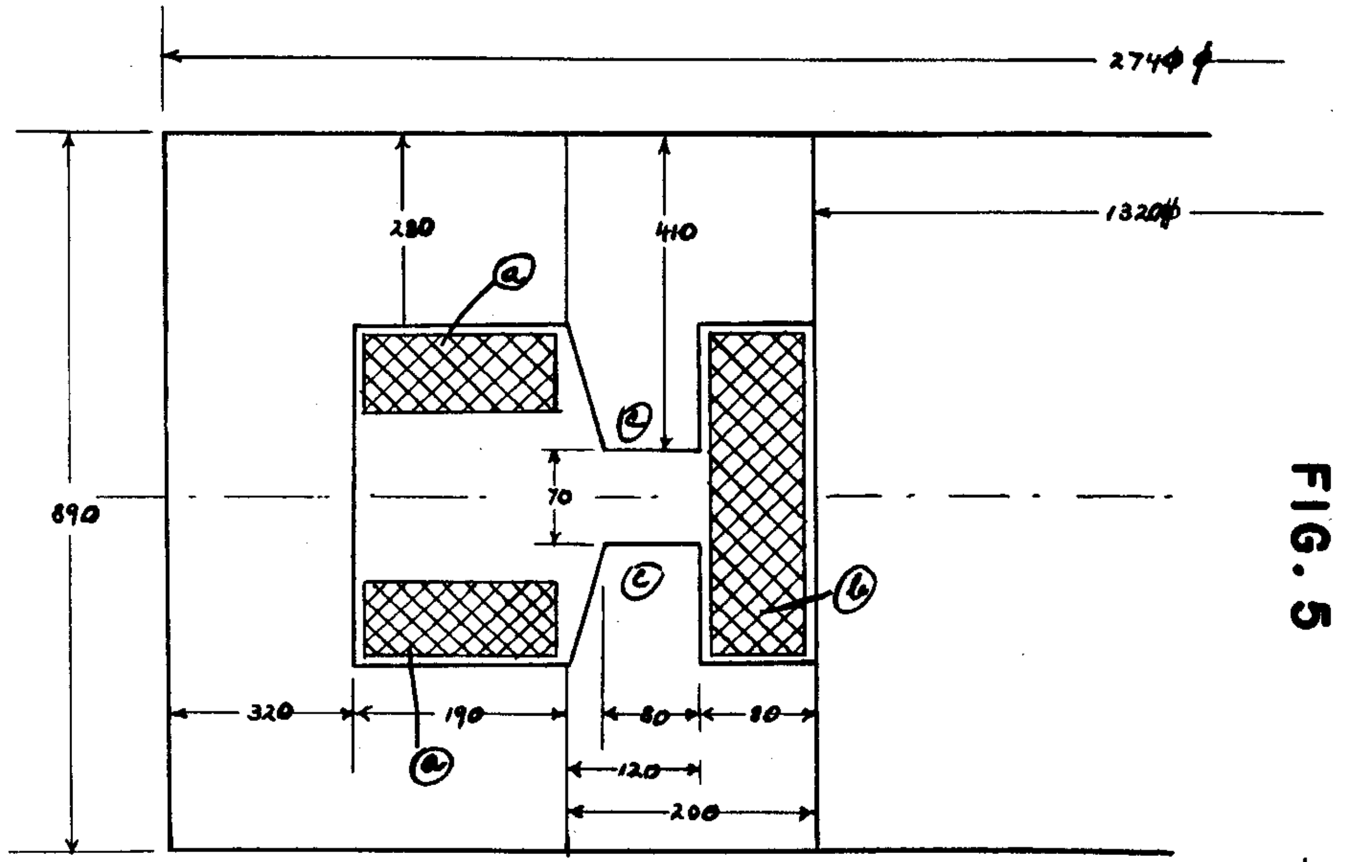
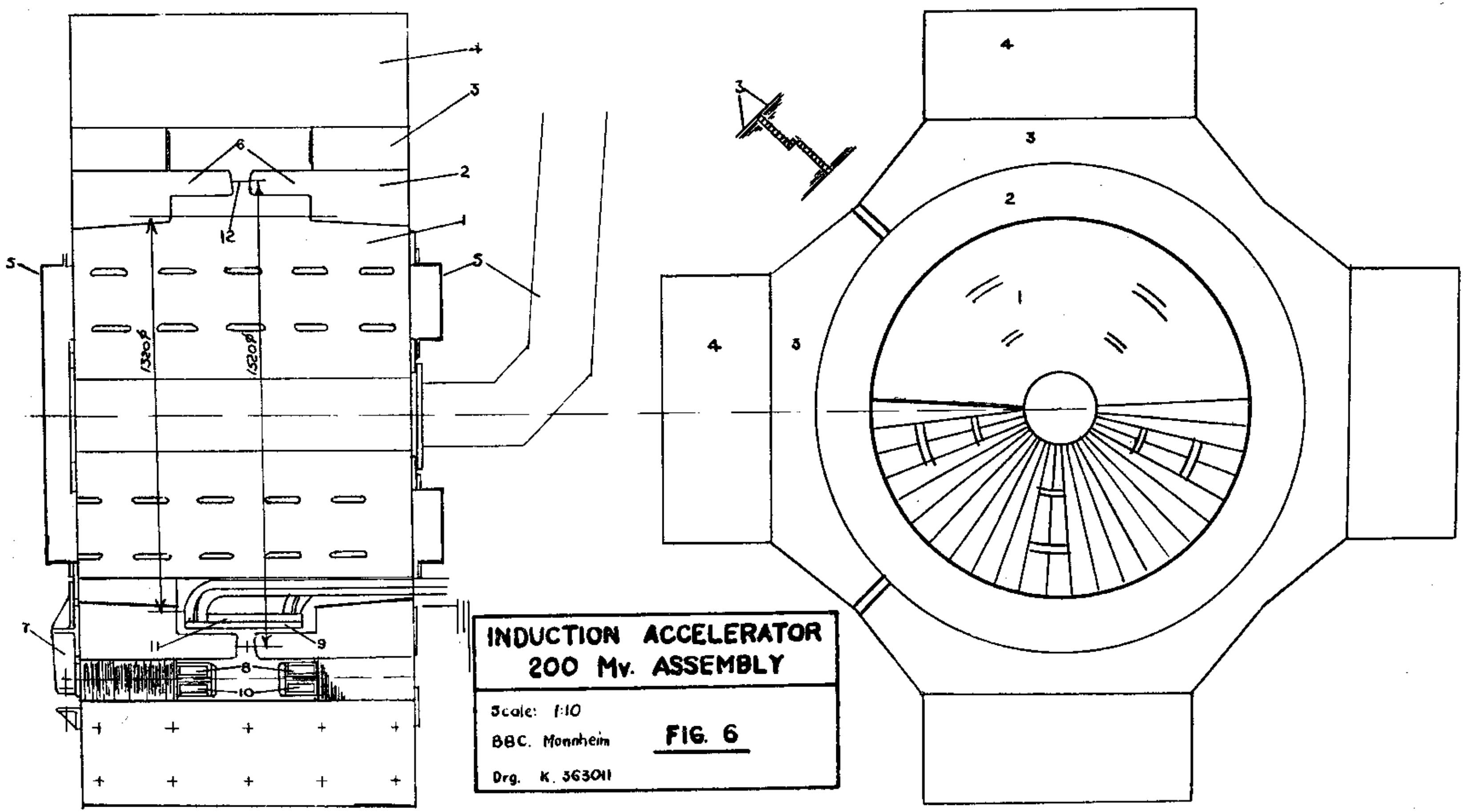


FIG. 5

Design for a 200 MV Betatron

Design for a 200 MV Betatron



INDUCTION ACCELERATOR
200 Mv. ASSEMBLY

Scale: 1:10

BBC. Mannheim

Drg. K. 363011

FIG. 6

DIAGRAM OF CONNECTIONS

200 Mv. ELECTRON

ACCELERATOR (Design)

FIG. 7

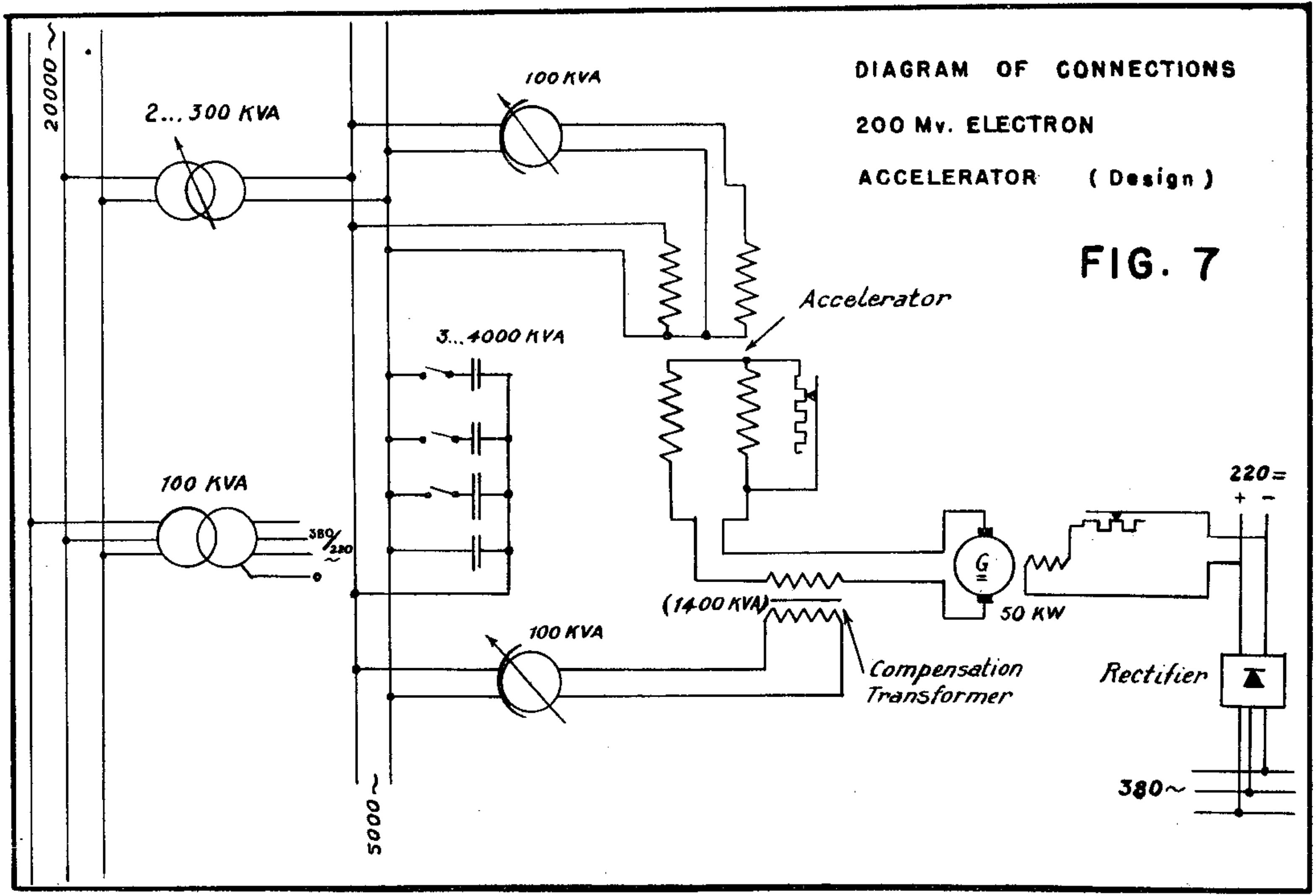
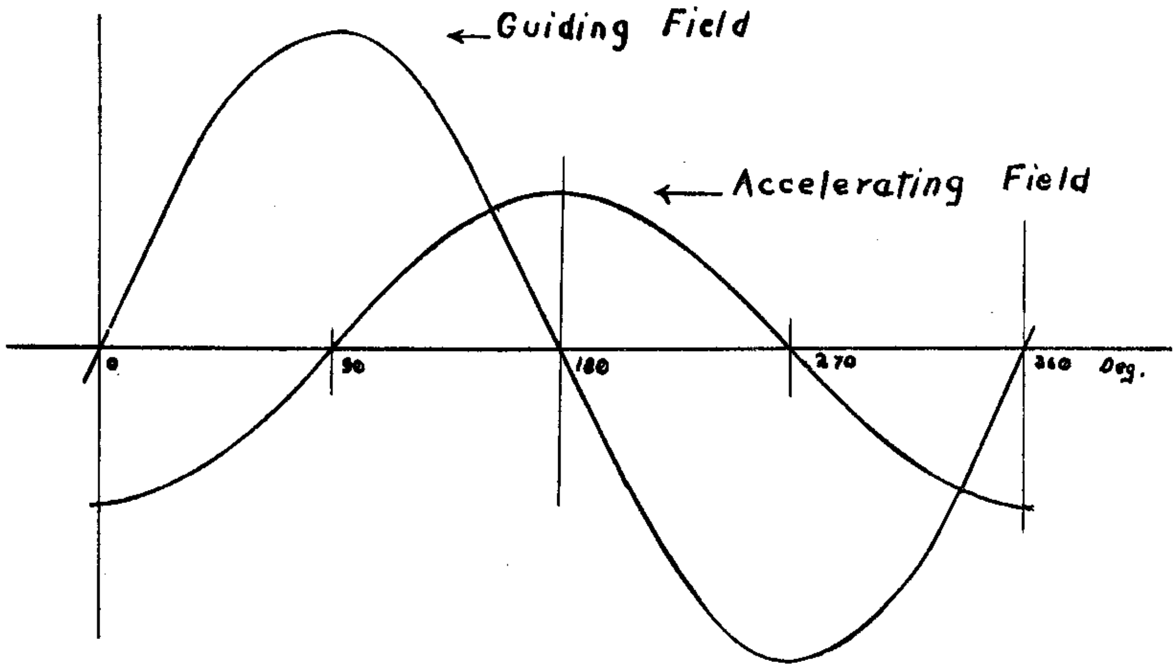


FIG. 8



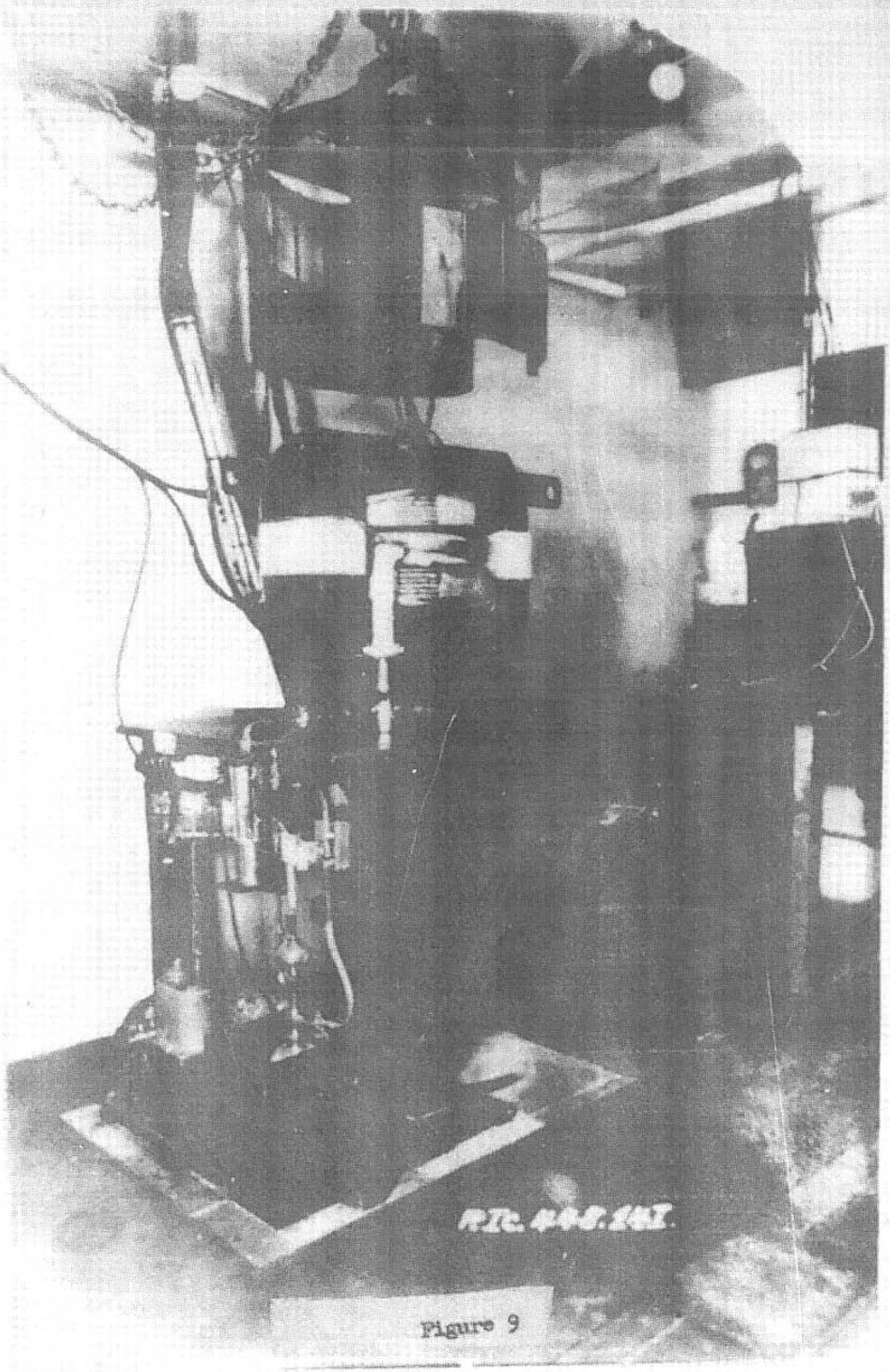


Figure 9

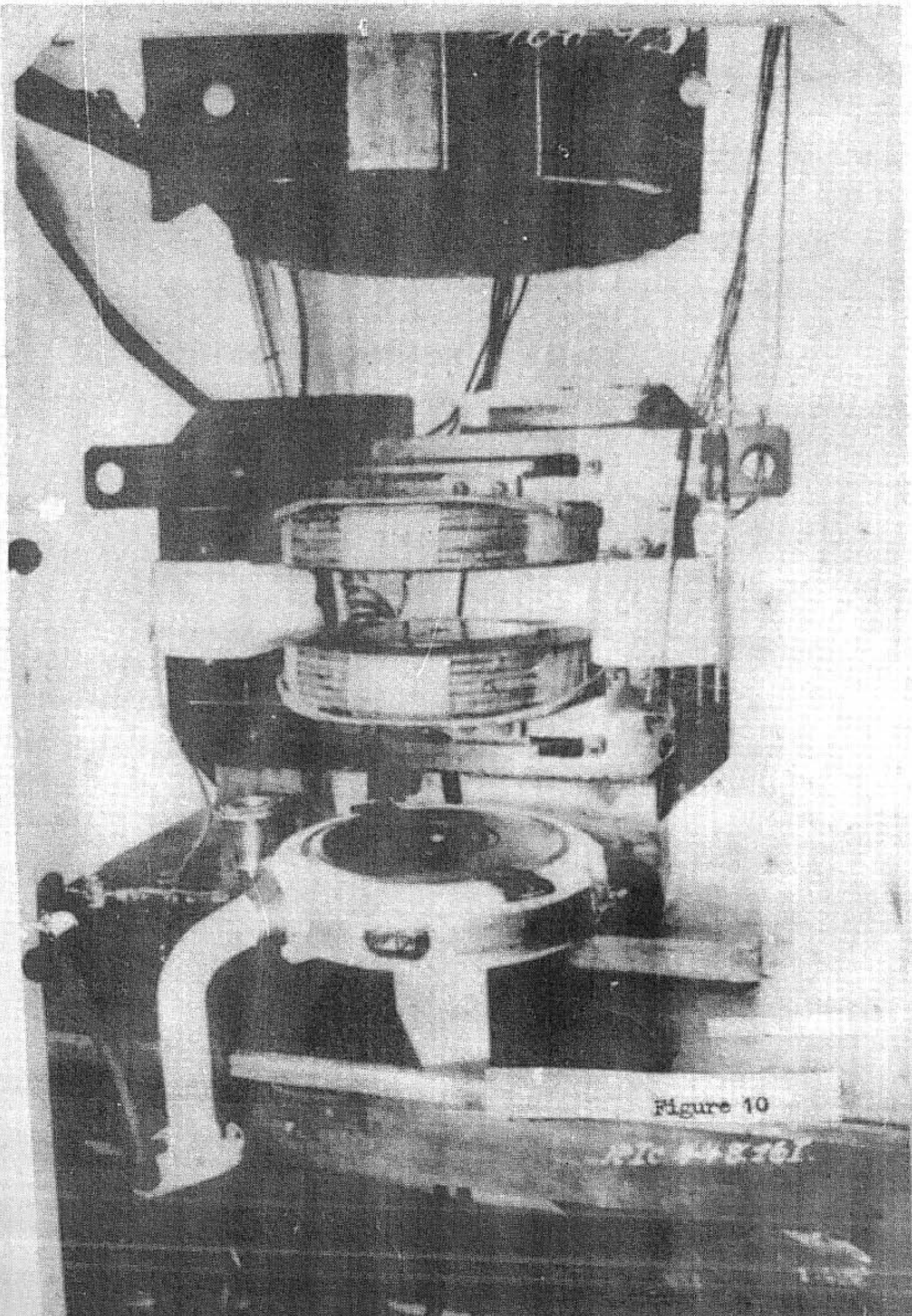
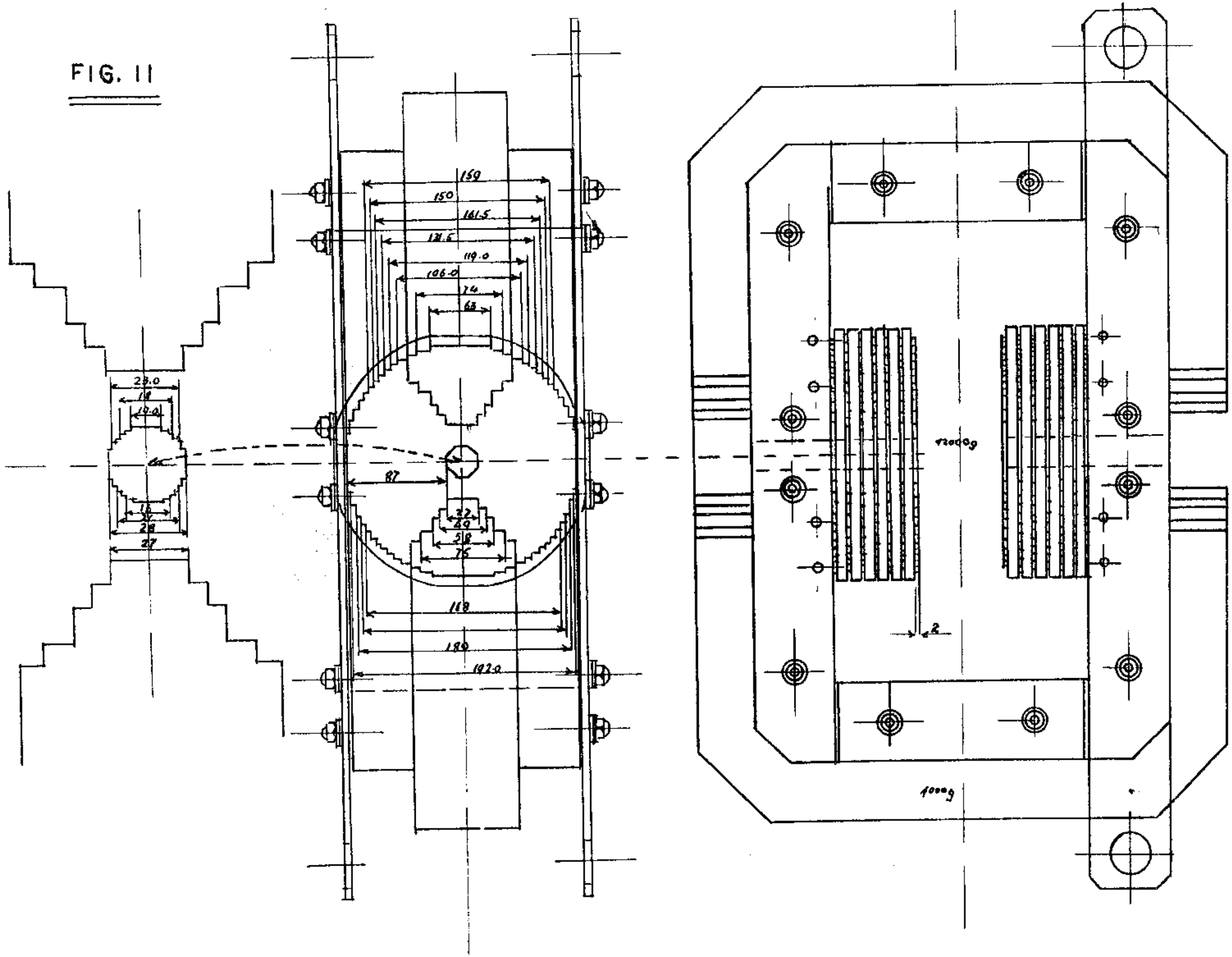


Figure 10

FIG. 11



23.0
18
10.0
16
28
27
159
150
161.5
131.5
119.0
106.0
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63
87
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49
58
75
168
180
192.0
120000g
2
1000g

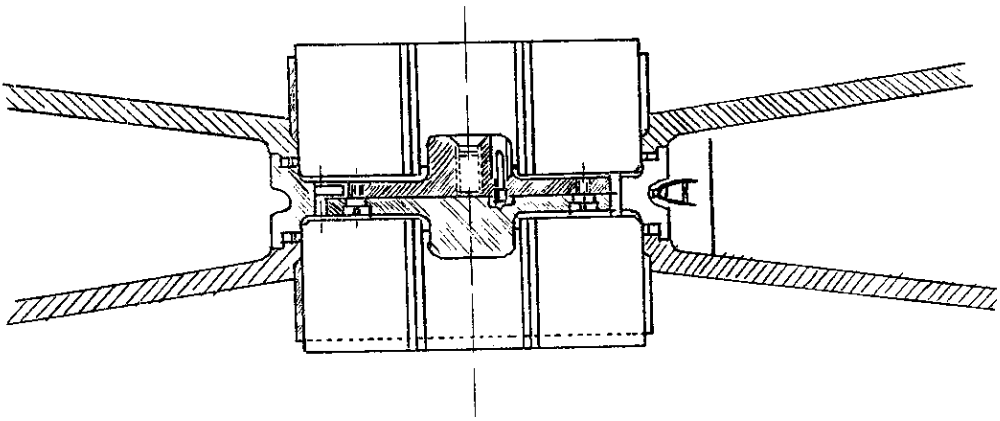

FIG. 12

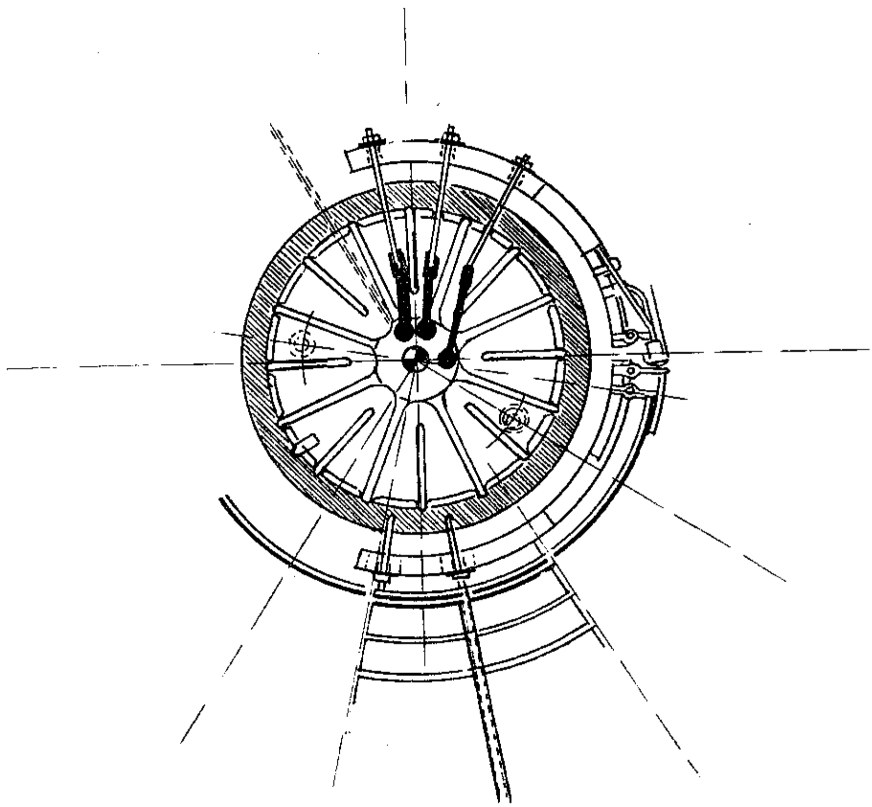

FIG. 13

DIAGRAM OF CONNECTIONS
5-6 Mv. Electron Accelerator
SIEMENS-REINIGER, Erlangen.